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Split Dimensional Regularization for the Temporal Gauge

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ABSTRACT

A split dimensional regularization, which was introduced for the Coulomb gauge by Leibbrandt and Williams, is used to regularize the spurious singularities of Yang-Mills theory in the temporal gauge. Typical one-loop split dimensionally regularized temporal gauge integrals, and hence the renormalization structure of the theory are shown to be the same as those calculated with some nonprincipal-value prescriptions.

In the studies of gauge theories, we are generally required to choose a gauge for quantization. Among the feasible gauge conditions, noncovariant gauges [1,2], which can be classified by constant four-vectors, have been most discussed from the technical point of view. Among the noncovariant gauges, the temporal gauge is known to be the most complicated and cumbersome. Nonprincipal value prescriptions [3,4] have been applied to study the renormalization of Yang-Mills theory quantized in the temporal gauge. These prescriptions provide useful calculational procedures for the dimensionally regularized temporal-gauge integrals. Recently, a regularization known as the split dimensional regularization was proposed by Leibbrandt and Williams [5] for Yang-Mills theory in the Coulomb gauge. In this brief report, we shall apply the split dimensional regularization to study the renormalization of Yang-Mills theory in the temporal gauge.

The temporal gauge is defined by the condition $n_\mu A^a_\mu = A^a_0 = 0$, where A^a_μ is the gauge potential and $n_\mu = (1,0,0,0)$ a constant temporal four-vector. In this gauge, the propagator has a spurious double pole at $q_0 = 0$ and reads $(i,j = 1,2,3)$

$$G^{ab}_{ij}(q) = \frac{i\delta^{ab}}{q^2 + i\epsilon}\left\{\delta_{ij} + \frac{q_i q_j}{q_0^2}\right\},\ G^{ab}_{0i}(q) = G^{ab}_{i0}(q) = G^{ab}_{00}(q) = 0, \tag{1}$$

or, in covariant form,

$$G^{ab}_{\mu\nu}(q) = \frac{i\delta^{ab}}{q^2 + i\epsilon}\left\{-\delta_{\mu\nu} + \frac{(q_\mu n_\nu + q_\nu n_\mu)}{q\cdot n} - \frac{q_\mu q_\nu}{(q\cdot n)^2}\right\}\ ,\ \epsilon > 0\,, \tag{2}$$

where we use a $(+,-,-,-)$ metric. We note that propagator (2) has a simple pole and a double pole at $q_0 = 0$.

Our purpose of this letter is to outline the calculational procedure for the one-loop regularized temporal-gauge integrals and to observe the gauge divergence problem of temporal-gauge theories. At the one-loop level, we shall need a regularization for regularizing the gauge divergences of the integrals. For the usual ultraviolet divergences that we are interested in, we employ split dimensional regularization with complex space-time dimensionality $2(\omega+\sigma) \equiv D$, with 2ω space dimensions and 2σ time dimensions.

We first consider the following integral with the spurious simple pole in Euclidean space:

$$I = \int \frac{d^D q}{(p-q)^2 q\cdot n}\,. \tag{3}$$

Using Feynman's parameterization and exponentiation formulae, we get

$$I = i \int_0^1 dx \int \frac{d^D q \quad q_0}{\left[x\left((p-q)^2 + i\epsilon\right) + (1-x) q_0^2\right]^2}$$

$$= i \int_0^1 dx \int d^D q \, \frac{q_0}{\left[q_0^2 + x\vec{q}^2 + xp^2 - 2xp_0 q_0 - 2x\vec{p}\cdot\vec{q}\right]^2}$$

$$= i \int_0^1 dx \int_0^\infty d\alpha \alpha e^{-\alpha x p^2} \int d^{2\omega} \vec{q} e^{-\alpha(x\vec{q}^2 - 2x\vec{p}\cdot\vec{q})} \int d^{2\sigma} q_0 q_0 e^{-\alpha(q_0^2 - 2xp_0 q_0)} \,. \tag{4}$$

Because

$$\int d^{2\omega} \vec{q} e^{-\alpha x(\vec{q}^2 - 2\vec{p}\cdot\vec{q})} = \pi^\omega (\alpha x)^{-\omega} e^{\alpha x \vec{p}^2} \,, \tag{5}$$

$$\int d^{2\sigma} q_0 q_0 e^{-\alpha(q_0^2 - 2xp_0 q_0)} = \left[\frac{\pi^{\sigma+\frac{1}{2}}(2\sigma-1)!!}{\Gamma(\sigma) 2^\sigma \alpha^{\sigma+\frac{1}{2}}} + xp_0 \pi^\sigma \alpha^{-\sigma}\right] e^{\alpha x^2 p_0^2} \,, \tag{6}$$

we obtain

$$I = i\pi^{\omega+\sigma} p_0 \int_0^1 dx x^{1-\omega} \int_0^\infty d\alpha \alpha^{1-\omega-\sigma} e^{-\alpha x(1-x) p_0^2}$$

$$= i\pi^{\omega+\sigma} p_0 \Gamma(2-\omega-\sigma) \int_0^1 dx x^{1-\omega}$$

$$= \frac{2p\cdot n}{n^2} \overline{I} \ , \ \ \overline{I} \equiv i\pi^2 \Gamma(2-\omega-\sigma) \ , \ \omega \to \frac{3}{2}, \ \sigma \to \frac{1}{2} \,. \tag{7}$$

This result is the same as that for the corresponding temporal gauge integral calculated with some nonprincipal-value prescriptions.

Next we turn to an integral with the double pole in Euclidean space:

$$J = \int \frac{d^D q}{(p-q)^2 (q\cdot n)^2} \,. \tag{8}$$

Following the same procedure for the previous integral, we get

$$J = i \int_0^1 dx \int \frac{d^D q}{\left[x\left((p-q)^2 + i\epsilon\right) + (1-x) q_0^2\right]^2}$$

$$= i \int_0^1 dx \int d^D q \, \frac{1}{\left[q_0^2 + x\vec{q}^2 + xp^2 - 2xp_0 q_0 - 2x\vec{p}\cdot\vec{q}\right]^2}$$

$$= i \int_0^1 dx \int_0^\infty d\alpha \alpha e^{-\alpha x p^2} \int d^{2\omega}\vec{q} e^{-\alpha(x\vec{q}^2 - 2x\vec{p}\cdot\vec{q})} \int d^{2\sigma} q_0 e^{-\alpha(q_0^2 - 2xp_0 q_0)} \,. \tag{9}$$

Performing the q-integral, we obtain

$$J = i\pi^{\omega+\sigma} \int_0^1 dx x^{-\omega} \int_0^\infty d\alpha \alpha^{1-\omega-\sigma} e^{-\alpha x(1-x)p_0^2}$$

$$= i\pi^{\omega+\sigma}\Gamma(2-\omega-\sigma) \int_0^1 dx x^{-\omega}$$

$$= \frac{-2}{n^2}\overline{I} \,, \omega \to \frac{3}{2}, \, \sigma \to \frac{1}{2} \,. \tag{10}$$

which is the same as the corresponding temporal integral calculated with some prescriptions [3,4]. Other temporal-gauge integrals needed for the one-loop gluon self-energy can be easily calculated (cf. ref. [4]).

We next briefly mention the renormalization of the temporal gauge theory in split dimensional regularization. We consider the one-loop gluon self-energy. Let the time-translation invariant gauge propagator [4] be:

$$G_{\mu\nu}^{ab}(q) = \frac{i\delta^{ab}}{q^2 + i\epsilon} \left(-\delta_{\mu\nu} + a_\mu(q) q_\nu - a_\nu(-q) q_\mu\right) \,, \tag{11}$$

where $a_\mu(q)$ is an arbitrary function related to the gauge choice. Let the ghost-gluon-ghost vertex be:

$$\Gamma_\mu^{abc}(q) = -g f^{abc} \left(\left[(a\cdot q) - 1\right] q_\mu - q^2 a_\mu(q)\right) \,, \tag{12}$$

and let the ghost propagator be:

$$G(q) = \frac{-i}{q^2 + i\epsilon} \,, \; \epsilon > 0 \,, \tag{13}$$

with g being the coupling constant, q_μ being the outgoing ghost's momentum. We get for

the temporal gauge

$$a_\mu(q) = \frac{n_\mu}{q \cdot n} - \frac{q_\mu}{2(q \cdot n)^2} , \tag{14}$$

$$\Gamma_\mu^{abc}(q) = g f^{abc} \frac{q^2 n_\mu}{q \cdot n} . \tag{15}$$

obtained by the Faddeev-Popov gauge-fixing procedure. Using the procedure, we have a ghost-gluon-ghost vertex that is proportional to n_μ and a ghost propagator that is inversely proportional to $q{\cdot}n$. It is easy to show that in split dimensional regularization the one-loop ghost diagram vanishes. Therefore, the calculation of the one-loop gluon self-energy requires considering one diagram with an internal gluon loop.

The calculation of the divergent part of the one-loop gluon self-energy has been carried out and yields

$$i\Pi_{\mu\nu}^{ab}(p) = \frac{11 g^2 C_A}{3(2\pi)^{2(\omega+\sigma)}} \delta^{ab} (p^2 \delta_{\mu\nu} - p_\mu p_\nu) \overline{I} , \tag{16}$$

where $C_A = N$ for $SU(N)$ gauge group. We observe that the self-energy is transverse and independent of the temporal vector n_μ. Thus, the renormalization structure in this method is the same as that in the temporal gauge calculated with some nonprincipal-value prescription.

In this work we have studied the renormalization structure of Yang-Mills theory by the split dimensional regularization. In this method, the dimensionality of the space component is $2\omega(\omega \to \frac{3}{2})$ and that of the time component is $2\sigma(\sigma \to \frac{1}{2})$. By using split dimensional regularization, we have shown that the results of integrals are the same as those with some nonprincipal-value prescriptions, but this method is seen to be considerably more straightforward.

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